THE AMERICAN
JOURNAL OF PSYCHOLOGY
Founded by G. Stanley Hall in 1887

Vol. XXVIII OCTOBER, 1917 No. 4

A CHART OF THE PSYCHOMETRIC FUNCTION

By Edwin G. Boring

In a drill course in the psychophysical methods the instructor has often to face the problem of making the form of certain mathematical functions intelligible to students who have only the competence afforded by the ill-remembered contents of a high-school algebra. In the Cornell Laboratory we have utilized the principle of the 'coin problem'[1] to construct a chart, which (1) accounts for the form of the curve of error, (2) indicates the manner in which the *phi*-function of *gamma* depends upon the curve of error, and (3) shows, in the light of these derivations, why the Φ (γ)-hypothesis provides a plausible equation for the psychometric function,—all without the use of more mathematics than the simplest algebra. It is true that some arbitrary assumptions are necessary, but they do not, we believe, offset the pedagogical gain of simplicity. We reproduce the chart herewith. Its exposition is as follows:

Let us take as our typical problem the determination of the limen of dual impression upon the skin. The original formulation of this problem (Fechner) required that we discover what value of the stimulus in cm. (separation of the aesthesiometer points) is necessary just barely to elicit the impression 'two.' In practice, however, very different values of stimulus sometimes give rise to the impression 'two' and sometimes not, so that there is no fixed point of change. We may nevertheless conceive as fixed *at any given moment* a point of

[1] See, for example, L. D. Weld, *Theory of Errors and Least Squares,* 1916, 41-51. To the unmathematical psychologist, who would nevertheless become psychophysicist, this book can not be too highly recommended!

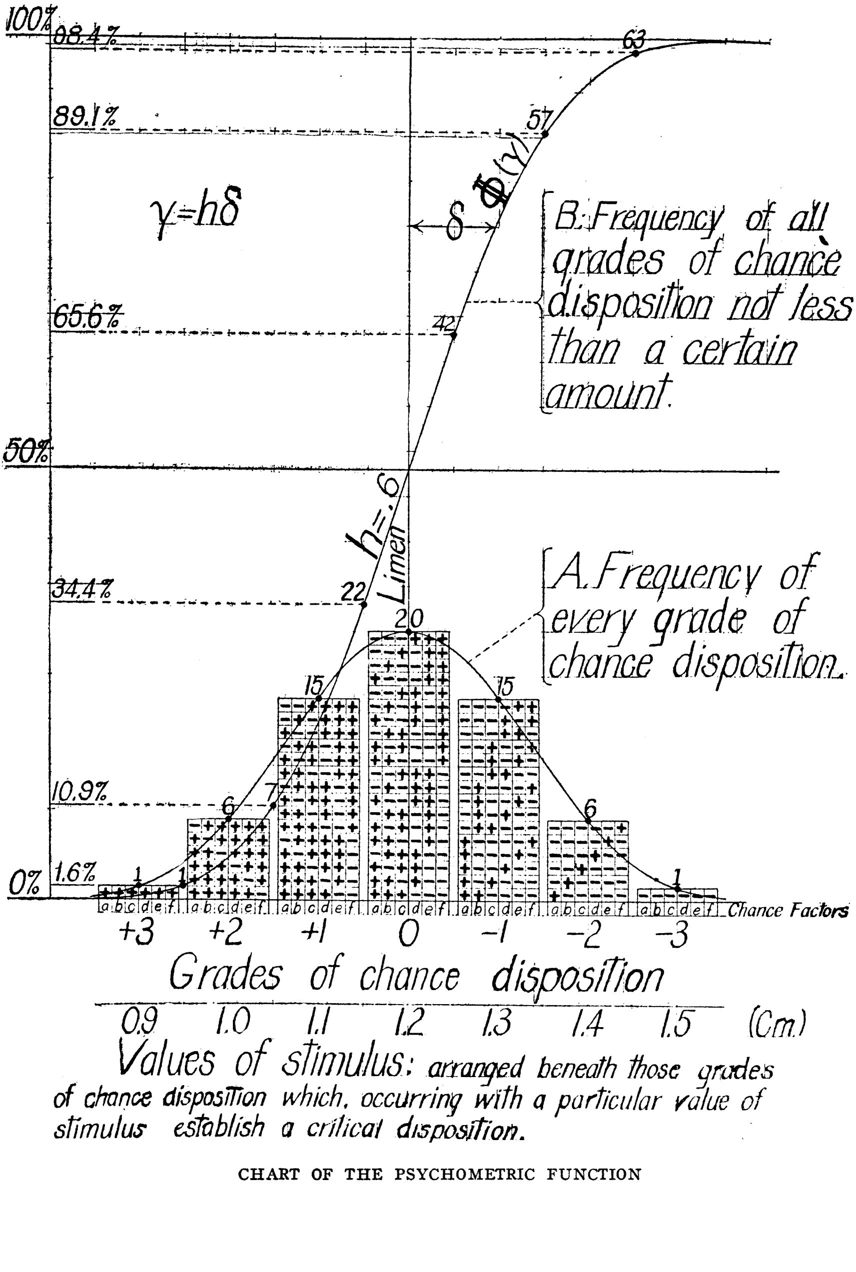

CHART OF THE PSYCHOMETRIC FUNCTION

change above which a stimulus would give the impression 'two' and below which it would not, provided that we do not forget the essential fact that this point is fixed only for the moment. The stimulated organ, we may then say, is variously disposed for impression,—a variation of disposition which occurs independently of the value of the stimulus, and as the result of the interplay of a large number of factors that work by chance, whenever the stimulus is applied, for or against the impression 'two.'

Ideally, in a psychophysical problem, we are concerned exclusively with a correlation between stimulus and impression. We presuppose an accurately controlled stimulus, an unvarying receptive apparatus the stimulation of which gives us the impression, and a mechanism that provides a report exactly reflecting this impression. Since, however, even in a careful experiment, we can control the stimulus only within the limitations of physical appliance, and the report only in so far as the observer can maintain constancy of attitude, we can never quite realize these ideal conditions. For the present, however, we shall overlook the possibility of departures from ideal conditions. We shall see, when we have finished our discussion, what part such departures may actually play in psychophysical work.

Let us suppose that there are, in connection with the organ of sense, just six factors, every one of which may dispose either toward or against the occurrence of the impression 'two' and therefore of the judgment 'two;' and let us call them *a, b, c, d, e,* and *f*. Further, let us represent a disposition toward the impression 'two' by "+" and a disposition against 'two' by "—". We assume that every factor disposes either toward or against the impression 'two' and never operates intermediately; and that the occurrence of "+" or "—" is independent of the stimulus and determined entirely by chance.[2] We have the possibility of seven grades of total disposition depending upon the chance occurrence of *pluses* and *minuses* for the six factors. (1) There is a single possibility that all six factors will dispose toward the impression 'two.' Let us call this grade of chance disposition "+3." It is indicated by the six concurrent *pluses* over the abscissa-value for "+3" on the chart. (2) There are six possible cases in every one of

[2]Actually we should need to assume an infinite number of factors or an infinite number of degrees of disposition; but we must keep our numbers finite or we shall complicate exposition.

If one wishes to think of "—" as the absence of a positive disposition rather than as a negative disposition, the argument is not altered.

which are five *pluses* and one *minus*,—a grade of chance disposition next less favorable to the 'two'-impression ("+2" on the chart). (3) There are fifteen possible ways in which four factors may dispose for and two against the dual impression, a chance disposition of "+1" (see chart). (4) The dispositional condition called "0" on the chart occurs when three factors dispose toward 'two' and three against; there are twenty such possible cases. When the *minuses* predominate, we get, in like manner, (5) fifteen cases for the negative disposition (against the impression 'two') of "—1," (6) six cases for "—2," and (7) a single case of maximal negative disposition, "—3." If we join these points we get the curve of error, which is marked "A" on the chart, and which gives the frequency of occurrence of every grade of chance disposition.[3] The familiar bell-shape of the curve represents the fact that there are more possible combinations of the chance factors when these are half and half unlike in effect than when they are all alike; and, further, that, in the region midway between these two conditions, the number of possible combinations decreases most rapidly.

The curve "B" is derived from "A". It shows the frequency of occurrence of all grades of chance disposition not less than (equal to or greater than) certain amounts. For instance, for the disposition called "+3" there is only the single case of the six *pluses* in which the stimulated organ is disposed for the impression 'two' by an amount not less than "+3." There are seven cases in which the dispositional grade is not less than "+2" (six in which it equals "+2" and one in which it is greater); there are 22 cases in which it is not less than "+1" (15 in which it is equal, 6 + 1 in which it is greater); and so on, with 42 cases (20 + 15 + 6 + 1) in which it is not less than "0," and 57, 63, and 64 cases in which it is not less than "—1," "—2," and "—3" respectively. Thus every point on the B-curve is the sum of all cases which lie to the left of it, as represented under the A-curve. The total number of cases is 64. If every sum is expressed as a percentage of the total number of cases we get the ordinate-scale at the left of the chart: 0, 1.6, 10.9, 34.4, 65.6, 89.1, 98.4, and 100%.

This B-curve is symmetrical about the 50%-line and would, if we had not arbitrarily taken a finite number of cases, be asymptotic to the 100% and 0% lines. It is known as the *phi*-function of *gamma*, $\Phi(\gamma)$. γ is a general unit of abscissa defined as the product $h\delta$, in which h is the measure of pre-

[3] Thus we may compute the ordinates by a formula for permutations; Weld, *op. cit.*, 42.

cision of both the A- and B-curves, and δ is the distance of any point on either curve from the ordinate of symmetry (in this case, the ordinate for balanced dispositional factors, the zero-ordinate on the chart).[4]

So far we have dealt only with the factors which vary by chance. The B-curve is merely an expression for the frequency with which dispositions toward the impression 'two', not less than certain amounts, occur by chance. But the controlled value of the stimulus also disposes more or less toward the impression 'two.' This impression will actually occur whenever the controlled disposition, dependent upon the value of the stimulus, and the total disposition of the receptive organ, dependent upon chance, are together adequate to the impression 'two.' The less the separation between the aesthesiometer-points, the greater must be the chance disposition of the organ, if the impression 'two' is actually to occur.

Suppose now that 1.2 cm. in the stimulus is critically adequate[5] to the impression 'two' whenever it occurs with the grade of chance disposition which we have called "0." Let us write "1.2" under "0" on the chart. A lesser stimulus, 1.1 cm., would require a greater chance disposition to be critically adequate to the impression 'two.' Let us say that 1.1 cm. is critically adequate when it occurs with the grade of chance disposition called "+1," and let us write "1.1" beneath "+1" on the chart. Then we may expect that 1.0 cm. will be critically adequate whenever it occurs with a chance disposition of "+2"; and so on. Thus we write under every grade of chance disposition the value of the stimulus which, in concurrence with that grade of chance disposition, would attain critical adequacy to the impression 'two.'

It now appears that we shall get the impression 'two' for a given stimulus whenever the grade of chance disposition which renders the given stimulus critically adequate to that impression occurs, or whenever there is a still more favorable grade of chance disposition. The B-curve shows the frequency of grades of chance disposition not less than given amounts; that is to say, the B-curve gives, when taken in relation to the scale of stimulus-values, the frequency with which a grade of chance disposition adequate to the impression

[4] For the use of these terms in the method of constant stimuli, *cf.* E. G. Boring, Urban's Tables and the Method of Constant Stimuli, *Amer. J. Psychol.*, 28, 1917, 281-285.

[5] By the phrase "critically adequate" we mean, in this discussion, to indicate the value which is the point of change from inadequacy to adequacy. We do not intend the Fechnerian notion; the point that is "critically adequate" is neither adequate nor inadequate, but is the point of change.

'two' will occur; or, more simply, the frequency of the 'two'-impressions for any value of stimulus. Thus, if we were to apply to the skin at random these values of the stimulus and were to record the frequency of occurrence of the judgment 'two' (which, we have assumed, corresponds exactly to the frequency of the impression 'two'), we should expect this curve of frequencies to be identical with the curve of dispositional chance factors (the B-curve), provided, of course, that our assumptions have been correct. The B-curve is, as we have seen, the Φ (γ); and a curve that gives frequencies of judgment as a function of the magnitude of stimulus is known as a *psychometric function*. Hence it follows that, under our assumptions, the Φ (γ) is the psychometric function.

We are now in a position to get some notion of what is meant by a *limen*. In practice, when we obtain empirically a set of data which approximate the Φ (γ) (the B-curve), we take the stimulus-value corresponding to the 50%-point of this curve as the limen. This is also the maximal point of the curve of error (the A-curve). The limen is, therefore, the value of stimulus which is critically adequate to the impression 'two' (hence the judgment 'two') whenever it occurs with the most frequent value of chance disposition; or, more simply, the limen is the stimulus which is most frequently critically adequate to the impression 'two.' Furthermore, we may note that the most frequent chance dispositional condition thus occurs when chance factors balance or cancel one another.

We began this discussion by making certain assumptions and we have argued that if these assumptions were valid the Φ (γ) would be the psychometric function. In practice, on the other hand, we find our psychometric functions empirically and then determine how closely they approximate the Φ (γ). We get, in practice, considerable agreement between hypothesis and fact; hence it appears that our assumptions are approximately valid.

Does this agreement between fact and theory also assure us that the value of the stimulus is accurately determined by the setting of the physical instrument and that the impression is exactly represented by the judgment? Not necessarily; it means that the errors which occur in either of these ways are relatively small, or are constant, or, perhaps, are due to chance. If small, they alter the limen by a negligible amount; if constant, they alter it by a fixed amount; if due to chance, they merge with the other chance factors and do not change the limen at all, although they may decrease its precision.